

## \* (5.2) Free Electron Theory \* of Metals

There are three main theories was explained about metal, they are :-

1. classical free electron theory (or) Lorentz and Drude
2. quantum free electron (Sommerfeld) theory (or) Sommerfeld free electron theory (1928).
3. Band theory of solids (or) zone theory (1928).

### 1. classical free electron theory :-

→ The classical free electron theory was explained by Lorentz & Drude in the year 1900.

→ It is also called free electron gas model.

postulates :-

→ The metals are made up of atoms, which contains large no. of free electrons.

→ The collisions b/w electron-electron and electron-nucleus are elastic collisions (or) negligible collisions.

i.e., there is no loss of energy due to collision.

→ The free electrons can move freely through out the metal in different directions with random velocities

i.e., the net velocity will be zero.

→ The free electrons can obey the properties of Maxwell - Boltzmann distribution of particles.

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→ The free electrons can obey the properties of kinetic theory of gases.

→ Here, the nucleus is treated as "positive ion core".

→ If the electric field is applied to the metal then the electrons can move opposite to the electric field.

→ Drift velocity :- ( $v_c$ )

The additional velocity of electron by the application of electric field is called drift velocity ( $v_c$ ).

→ Mean free path ( $\lambda$ ) :- The average path travelled by the electron in b/w two successive collisions is called mean free path.

→ Mean collision time ( $T_c$ ) :- The average time taken by the electron in b/w two successive collisions is called mean collision time.

→ Relaxation time ( $\tau$ ) :- The time taken by the electron to decay its velocity from disturbed position to initial position (equilibrium position) is called relaxation time of the electron.

→ Mobility ( $\mu$ ) :- The drift velocity per unit electric field intensity is called mobility of the electron.

$$\mu = \frac{V_d}{E}$$

→ If 'e' be the charge of the electron.

'n' be the concentration of electrons.

$V_d$  be the drift velocity of electron

then the current density can be represented by  $J = nev_d$   
 $J = ne\mu E$

Drawbacks of classical free electron theory:-

→ According to classical free electron theory, the electrical conductivity of metal is inversely proportional to the square root of absolute temperature.

i.e.,  $\sigma \propto \frac{1}{\sqrt{T}}$

But practically, the electrical conductivity of metal is inversely proportional to the absolute temperature only.

i.e.,  $\sigma \propto \frac{1}{T}$

→ According to classical free electron theory, the specific heat of solids ( $\theta$ ) is independent on the absolute temperature ( $T$ ).

which is not true, i.e.,  $\theta \propto T$

But practically, the specific heat of solid ( $\theta$ ) is directly proportional to the absolute temperature ( $T$ ).

i.e.,  $\theta \propto T$

→ According to classical free electron theory, the electrical conductivity of metal

$$\sigma = \frac{ne^2\tau}{m}$$

i.e., It explained the electrical conductivity of metals

But, this theory fails to explain the electrical conductivity of semiconductors & insulators.

→ The classical free electron theory, fails to explain the ferromagnetism of metals.

→ This theory, fails to explain

(i) photo electric effect:

(ii) Compton effect:

(iii) Black Body radiation.

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→ According to this theory the Weidmann-Franz law

i.e.  $\frac{k}{\sigma T} = \text{constant}$

But practically the value of  $\frac{k}{\sigma T}$  is not constant. It is also failure of classical free electron theory.

2. Quantum free electron theory:-

A quantum free electron theory was explained by

Sommerfeld in the year 1928.

postulates:-

→ The metals are made up of atoms which contains large number of free electrons.

→ The collisions b/w electron-electron and electron-nucleus are elastic collisions.

i.e., there is no loss of energy due to collision.

→ According to this theory, the free electrons can obey the properties of Fermi-Dirac statistics.

→ According to this theory, the metal is treated as a potential box in which the electron exhibits the discrete energy values.

→ The discrete energy values (or) quantised energy values of the electron can be represented by the equation

$$E_n = \frac{n^2 h^2}{8ma^2}$$

If  $n=1 \Rightarrow E_1 = \frac{1^2 h^2}{8ma^2}$

$$E_1 = \frac{h^2}{8ma^2}$$

which is called ground state energy (or) minimum energy

If  $n=2 \Rightarrow E_2 = \frac{8^2 h^2}{8ma^2}$

$E_2 = \frac{4h^2}{8ma^2}$

$E_2 = 4E_1 \rightarrow$  first excited state energy of the electron.

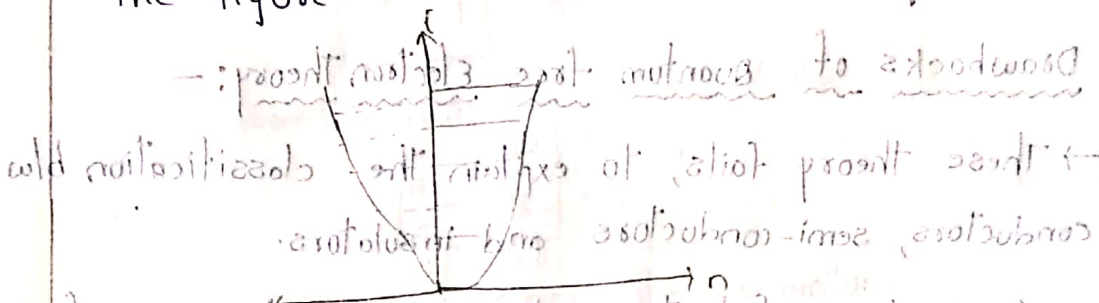
Similarly,  $n=3 \Rightarrow E_3 = 9E_1$

$E_4 = 16E_1$

$E_5 = 25E_1$

and  $\dots$  represents second, third, fourth  $\dots$  are excited state energy of the electron.

$\rightarrow$  The graph b/w the quantum number ( $n$ ) and the energy value ( $E$ ) represents a parabola as shown in the figure.

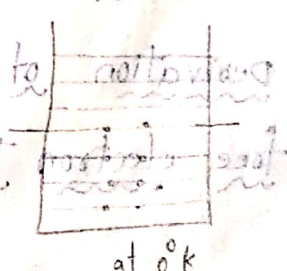


According to this theory, the distribution of electrons among the energy states with respect to Pauli's exclusive principle:

i.e.) Each energy state contains maximum two electrons with opposite spin and momenta.

$\rightarrow$  The highest energy state with filled electrons is called Fermi energy state ( $E_F$ ).

$\rightarrow$  The energy starting b/w  $n^{\text{th}}$  energy state and  $(n+1)^{\text{th}}$  energy state



$$(n+1)^2 E_1 - n^2 E_1 = (2n+1) E_1$$

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## Success of Quantum free electron theory:-

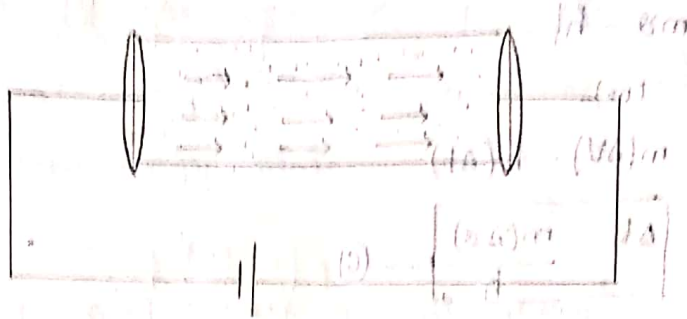
- It explains electrical and thermal conductivity of metals.
- It explains the thermionic emission.
- It explains the photoelectric effect, thermoelectric effect and gravitational effects of metals.
- It explains the paramagnetic susceptibility.
- The quantum free electron theory explain the specific heat behaviour of metals.
- It explains successfully the temperature dependent of electrical conductivity of metals.
- It successfully explain the dependence of electrical conductivity on electrical concentration.

## Drawbacks of Quantum free electron theory:-

- These theory fails, to explain the classification b/w conductors, semi-conductors and insulators.
- These theory fails to explain the negative sign of Hall co-efficient in the case of metals.
- These theory fails to explain electrical conductivity dependence on electron concentration in the case of di-valent and tri-valent metals.
- These theory fails to explain the concept of Fermi surface of metals.

## Derivation of electrical conductivity based on Quantum free electron theory:-

$$J = \sum_{\mathbf{k}} \mathbf{v}(\mathbf{k}) \cdot \mathbf{E} = -e \sum_{\mathbf{k}} \mathbf{v}(\mathbf{k}) \cdot \mathbf{E}$$



→ let us consider a metal which contains large no. of free electrons. When the electric field is applied to the metal, then the force acting on the electron whose charge is 'e' by the electric field intensity 'E' can be written as

$$F = eE \quad \text{--- (1)}$$

→ we know that the rate of change of momentum is called force

$$\frac{dp}{dt} = F \quad \text{--- (2)}$$

from (1) and (2)

$$\frac{dp}{dt} = eE \quad \text{--- (3)}$$

Now let us consider the momentum

$$P = \hbar k$$

Now differentiating above equation w.r.t to 't'.

$$\frac{dp}{dt} = \hbar \frac{dk}{dt} \quad \text{--- (4)}$$

from (3) & (4)

$$eE = \hbar \frac{dk}{dt}$$

$$\frac{dk}{dt} = \frac{eE}{\hbar}$$

$$dk = \frac{eE}{\hbar} (dt)$$

$$(\Delta k) = \frac{eE}{\hbar} (\Delta t) \quad \text{--- (5)}$$

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$$mv = \hbar k$$

(or)

$$m(\Delta v) = \hbar(\Delta k)$$

$$\Delta k = \frac{m(\Delta v)}{\hbar} \quad \text{--- (6)}$$

from (5) & (6)

$$\frac{m(\Delta v)}{\hbar} = \frac{eE}{\hbar} (\tau)$$

$$\Delta v = \frac{eE}{m} (\tau)$$

$$\Delta v = \left(\frac{e\tau}{m}\right) E \quad \text{--- (7)}$$

→ We know that, the current density

$$J = ne(\Delta v) \quad \text{--- (8)}$$

sub (7) in (8)

$$J = ne \left(\frac{e\tau}{m}\right) E$$

$$J = \left(\frac{ne^2\tau}{m}\right) E \quad \text{--- (9)}$$

→ The above equation can be represents the current density of metal.

We know that,  $J = \sigma E$  --- (10)

$$\sigma = \frac{J}{E}$$

$$\sigma = \left(\frac{ne^2\tau}{m}\right) E \times \frac{1}{E}$$

$$\sigma = \frac{ne^2\tau}{m}$$

→ The above equ<sup>n</sup> represents the electrical conductivity of metals.



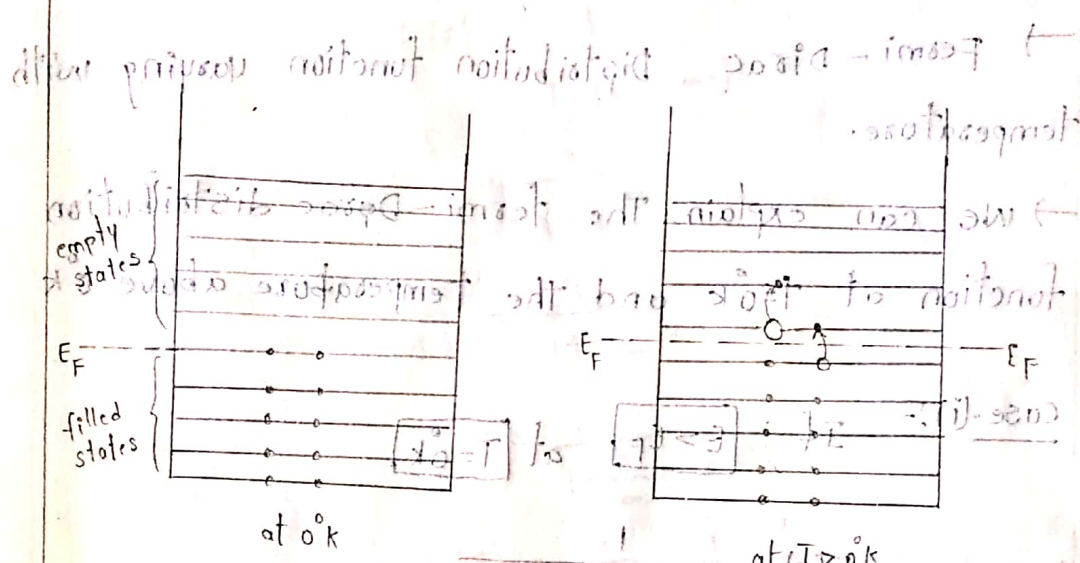
Fermi - Energy level (or) Fermi energy level :-

"The highest energy state with filled electrons at 0K is called fermi energy state."

"The energy states which can separates filled states and empty states at 0K is called fermi energy state."

The corresponding energy of the energy state is called fermi energy.

It is denoted by  $E_F$



The energy state at which the electron occupancy probability is  $\frac{1}{2}$  (or) 0.5 (or) 50% at any temperature above 0K is called Fermi Energy state.

\* → The fermi energy state is decreases with increasing temperature.

Fermi - Dirac distribution function :-

→ The Fermi - Dirac gave an equation to represents the electron occupancy probability among the energy states at 0K and above 0K which is called Fermi - Dirac distribution function (or) Fermi - Dirac probability function.

The Fermi-Dirac distribution function can be written as

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Where  
 $F(E) \rightarrow$  probability function  
 $E_F \rightarrow$  Fermi energy  
 $k \rightarrow$  Boltzmann's constant  
 $T \rightarrow$  Absolute temperature

$\rightarrow$  Fermi-Dirac distribution function varying with temperature.

$\rightarrow$  We can explain the Fermi-Dirac distribution function at  $T=0^\circ\text{K}$  and the temperature above  $0^\circ\text{K}$

Case-(i):- If  $E > E_F$  at  $T=0^\circ\text{K}$

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

at  $T=0^\circ\text{K}$ ,  $kT=0$ , so the denominator becomes  $1 + \exp\left(\frac{+ve}{0}\right) = 1 + \exp(\infty)$

$$F(E) = \frac{1}{1 + e^\infty}$$

Since  $e^\infty$  is a very large number,  $1 + e^\infty \approx e^\infty$ , so  $F(E) \approx \frac{1}{e^\infty} \approx 0$

$$F(E) = 0$$

i.e., the energy states which are above the Fermi energy state

are empty state.

Because, the electron occupancy probability is  $F(E)=0$  at  $0\text{K}$ .

Case (ii)

if  $E < E_F$  at  $T = 0\text{K}$

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$= \frac{1}{1 + \exp\left(\frac{-ve}{kT}\right)}$$

$$F(E) = \frac{1}{1 + \exp(-\infty)}$$

$$= \frac{1}{1 + e^{-\infty}}$$

$$= \frac{1}{1 + \frac{1}{\infty}}$$

$$= \frac{1}{1 + 0}$$

$$F(E) = \frac{1}{1+0} = \frac{1}{1} = 1$$

i.e., the energy states which are below the Fermi-energy states (00) complete state. because, the electron occupancy probability is  $F(E)=1$  at  $0\text{K}$

→ From, the above two cases the Fermi-energy state can be defined as the energy state which can separate the filled state and energy state at  $0\text{K}$ .

Case:-(iii)

if  $E = E_F$  at  $T > 0\text{K}$

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

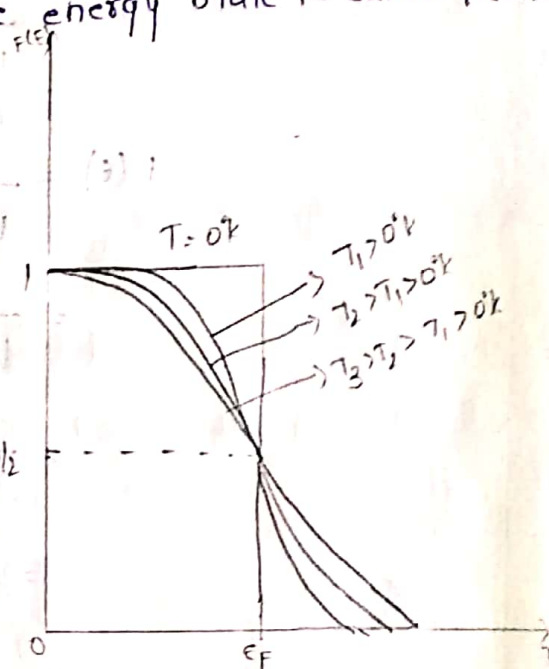
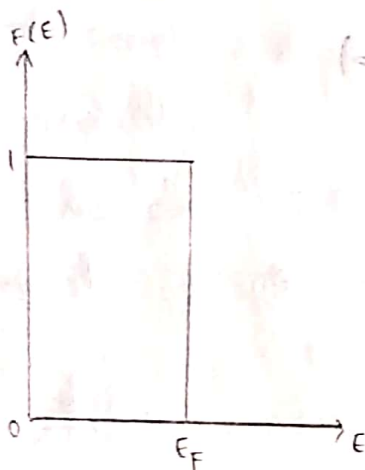
$$= \frac{1}{1 + \exp(\epsilon)}$$

$$F(\epsilon) = \frac{1}{1 + e^{\epsilon}}$$

$$= \frac{1}{1 + 1}$$

$$F(\epsilon) = \frac{1}{2}$$

i.e., the energy state at which the electron occupancy probability is '1/2' then the energy state is called Fermi-energy state.



Density of Energy states: -  $Z(\epsilon)d\epsilon$

"The no. of energy states per unit volume is called density of energy states"

We know that, the no. of energy states in the energy interval  $d(\epsilon)$  it can be written as

$$Z(\epsilon)d\epsilon = \frac{\pi}{2} n^2 dn \rightarrow \textcircled{1}$$

→ We know that, the quantised energy state of electron

$$E = \frac{n^2 h^2}{8ma^2}$$

$$n^2 = \left( \frac{8ma^2}{h^2} \right) E \rightarrow \textcircled{2}$$

$$n = \left( \frac{8ma^2}{h^2} \right)^{1/2} E^{1/2} \quad \text{--- (3)}$$

Now Differentiating Equ (3) both sides

$$2n \, dn = \left( \frac{8ma^2}{h^2} \right) dE$$

$$dn = \frac{1}{2n} \left( \frac{8ma^2}{h^2} \right) dE$$

$$dn = \frac{1}{2 \left( \frac{8ma^2}{h^2} \right)^{1/2}} \left( \frac{8ma^2}{h^2} \right) dE$$

$$dn = \frac{1}{2} \left( \frac{8ma^2}{h^2} \right)^{1/2} E^{-1/2} dE \quad \text{--- (4)}$$

sub Equ (3) & (4) in (1)

$$z(E) dE = \frac{\pi}{2} \left( \frac{8ma^2}{h^2} \right)^{1/2} E \left[ \frac{1}{2} \left( \frac{8ma^2}{h^2} \right)^{1/2} E^{-1/2} dE \right]$$

$$z(E) dE = \frac{\pi}{4} \left( \frac{8ma^2}{h^2} \right)^{3/2} E^{1/2} dE \quad \text{--- (5)}$$

→ According to Pauli's Exclusive principle, Each energy state will contain 2 electrons.

∴ The above equation can be written as

$$z(E) dE = 2 \times \frac{\pi}{4} \left( \frac{8ma^2}{h^2} \right)^{3/2} E^{1/2} dE$$

$$z(E) dE = \frac{\pi}{2} \left( \frac{8ma^2}{h^2} \right)^{3/2} E^{1/2} dE$$

$$= \frac{\pi}{2} \frac{(8)^{3/2} (m)^{3/2} (a^2)^{3/2}}{(h^2)^{3/2}} E^{1/2} dE$$

$$z(E) dE = \frac{4\pi}{h^3} (2m)^{3/2} \left( \frac{a^3}{3} \right) E^{1/2} dE \quad \text{--- (6)}$$

→ The above equation represents, the no. of energy states  $dE$  in the above interval  $dE$  (56)

∴ The no. of energy states per unit volume ( $a^3$ ) is called

density of energy states:  $\rho(E) = \frac{dN}{dE}$

→ The density of energy states can be written as

$$Z(E)dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE \quad (\because a=1)$$

Questions:-

1. Explain the drawbacks of classical free electron theory?
2. Explain the quantum free electron theory (success, drawbacks, postulates)?
3. Derive the equation for electrical conductivity. Based on quantum free electron theory?
4. What is Fermi energy (or) Fermi energy state? Explain the Fermi-Dirac distribution function varies with temperature?
5. Define & derive density of energy states?

$$Z(E)dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE$$

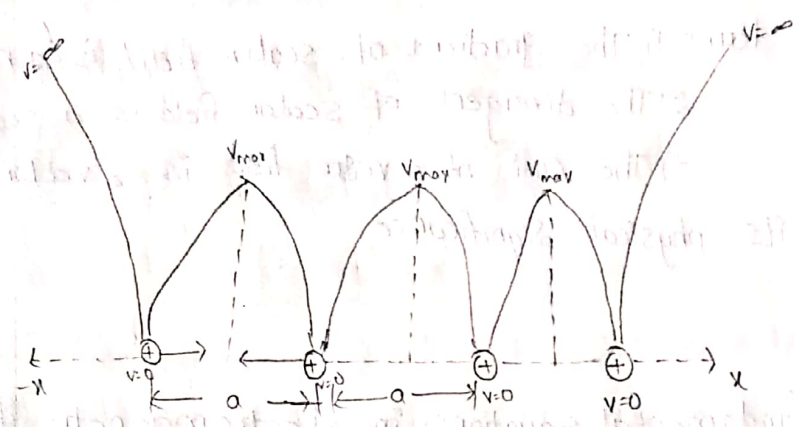
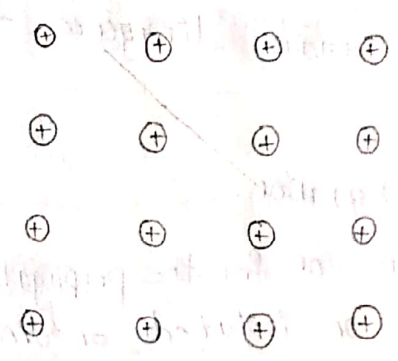
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\* 6.1 Band theory of solids \* with period function

Bloch theorem:-



→ According to Bloch the potential varies with the periodicity 'a'

i.e., the potential is minimum at the +vely charge nuclei as

$$V_x = 0$$

→ And in between the two positively charged nuclei the potential  $v$  is maximum

i.e.  $V_x = V_{max}$

→ According to Bloch the potential  $v(x)$  is varies as  $v(x+ta)$

i.e.,  $v(x) = v(x+ta)$

where  $a$  is the periodicity

→ Generally, the motion of the electron in the metal can be represented by a fundamental wave eqn

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (\epsilon - V_0) \psi = 0$$

∴ The general solution of the above Schrödinger wave equation will be taken as

$$\psi(x) = e^{\pm ikx}$$

→ But, according to Bloch  $V(x) = V(x+a)$  then, the Schrödinger wave equation can be written as

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (\epsilon - V(x+a)) \psi = 0$$

Then, the general solution of above equation can be taken as

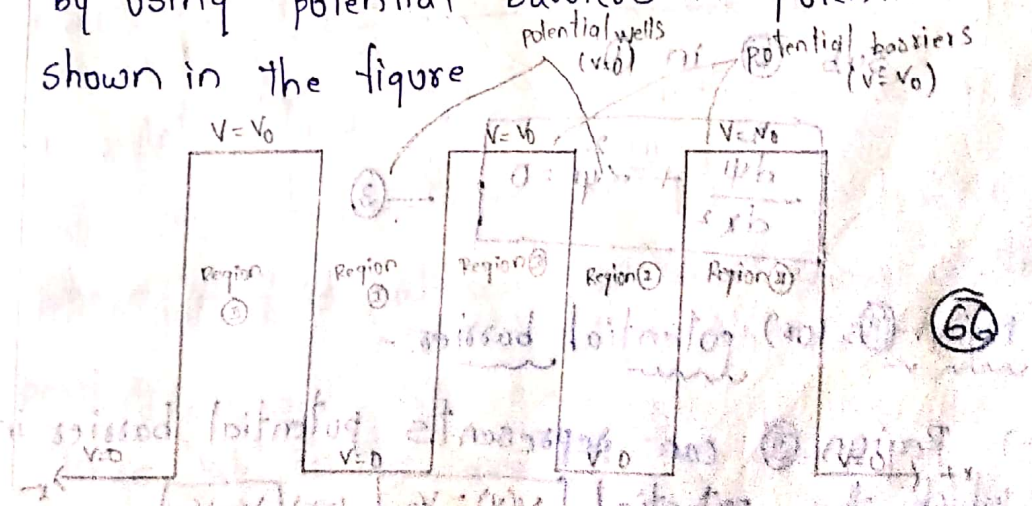
$$\psi(x) = u_k(x) e^{\pm ikx}$$

where,  $u_k(x) \rightarrow$  periodic function.

i.e., Bloch states that the plane wave solutions of the wave equations are modulating with the periodic functions.

Kronig-penney model (or) The periodic motion of electron in atomic lattice:

The periodic motion of electron in atomic lattice was explained by the scientist Kronig & Penney by using potential barriers and potential wells as shown in the figure





→ In the figure, the region (i) can represent potential wells in which the potential  $V=0$ .

→ In the figure, the region (ii) can represent potential barriers, the potential 'V' is maximum.

→ Let the maximum potential will be taken as 'V<sub>0</sub>'

$$V(x) = V_0$$

Region (i) (or) potential well:-

→ Region (i) can represent potential well in which the potential  $V=0$ .

→ Let us consider a particle which is present in potential well then the Schrödinger wave equation

can be written as

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

In potential  $V=0$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \text{--- (1)}$$

$$0 < x < a$$

→ Let  $\alpha^2 = \frac{2mE}{\hbar^2}$  --- (2)

sub (2) in (1)

$$\frac{d^2\psi}{dx^2} + \alpha^2 \psi = 0 \quad \text{--- (3)}$$

Region (ii) (or) potential barrier:-

→ Region (ii) can represent potential barrier in which the potential  $V(x) = V_0$  (or)  $V = V_0$

→ let us consider a particle which is present in potential barrier, then the schrodinger wave equation can be written as

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V_0) \psi = 0 \quad \text{--- (4)}$$

In potential barrier  $V = V_0$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V_0) \psi = 0 \quad \text{--- (5)}$$

→ let  $\beta^2 = \frac{2m}{\hbar^2} (E - V_0)$  --- (6)

sub (6) in (5)

$$\frac{d^2\psi}{dx^2} + \beta^2 \psi = 0 \quad \text{--- (7)}$$

→ According to Bloch the general solution of the above differential equation can be taken as

$$\psi = e^{\pm ik} u_2(x) \quad \text{--- (8)}$$

Now differentiate above equation twice w.r.t 'x' and these values substitute in equ. (3) & (7) then finally we will get the condition as

$$\frac{mv_0ab}{\hbar^2} \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \quad \text{--- (9)}$$

let  $p = \frac{mv_0ab}{\hbar^2}$  --- (10)

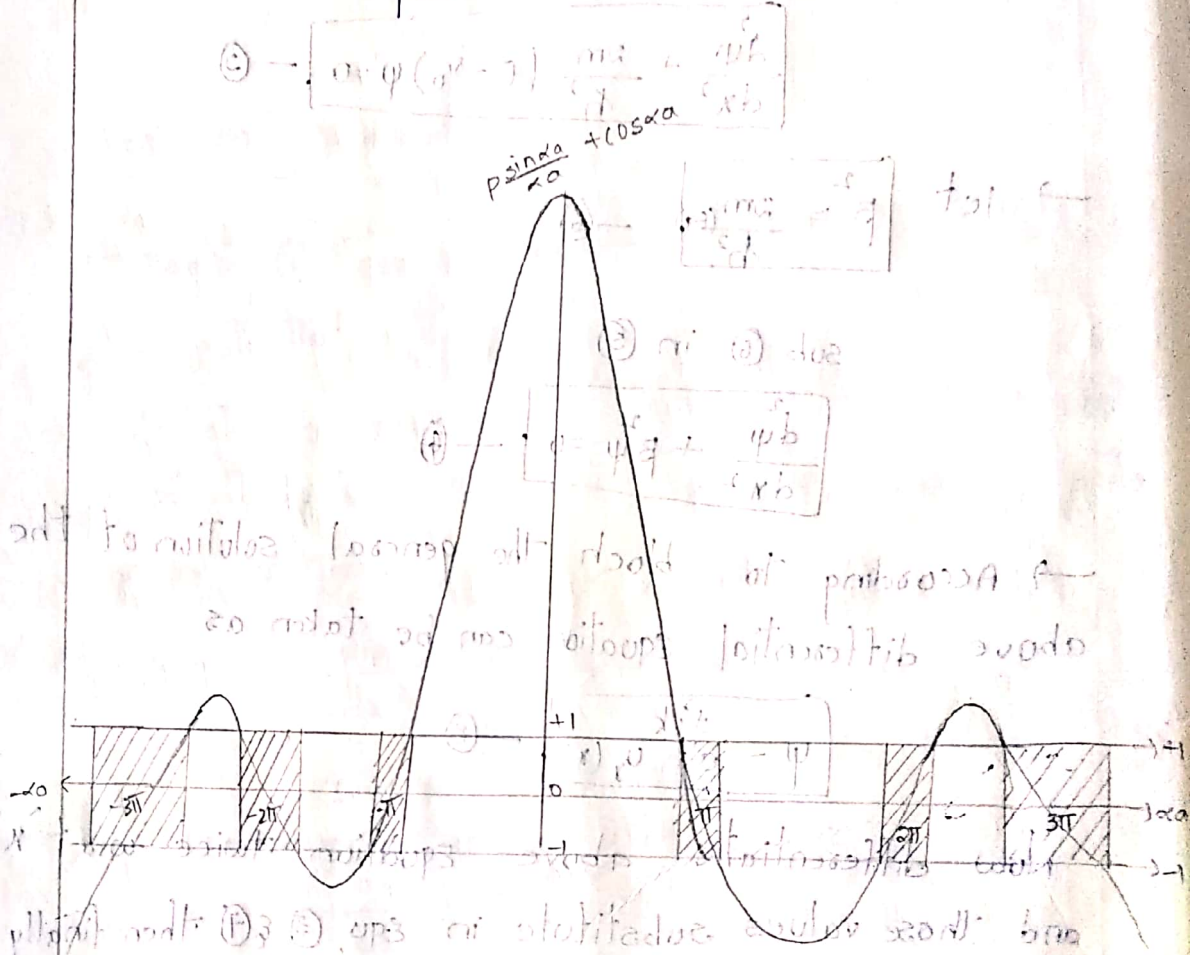
where 'p' is called scattering power of the barrier. (67)

Here 'v<sub>0</sub>b' is the area of the barrier.

Now sub (10) in (9) ...

$$p \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \quad \text{--- (11)}$$

→ Now plot a graph between  $p \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$  to  $\alpha a$  for a particular value  $p = \frac{3\pi}{2}$  then the graph is as shown in the figure.



→ from the above graph we conclude the following points.

1. The allowed bands of  $\alpha a$  are shown by shaded regions which are separated by forbidden bands.

i.e.,  $\left[ \alpha^2 = \frac{2mE}{\hbar^2} \right]$

2. If the value  $\alpha a$  increases then the width of the allowed band increases then the forbidden band decreases.

3. If the scattering power of the barrier  $p'$  increases then the width of the forbidden band increases then the allowed band decreases.

$$\therefore \left( p = \frac{m v_0 a b}{\hbar^2} \right)$$

4. If the scattering power of the barrier  $p \rightarrow \infty$  then the allowed band reduces and these will be appear as single lines as shown in the figure.

i.e.,  $\sin \alpha a = 0$

But we know that  $\sin n\pi = 0$

$$\alpha a = n\pi$$

$$\alpha = \frac{n\pi}{a}$$

$$\alpha^2 = \frac{n^2 \pi^2}{a^2}$$

where  $\alpha^2 = \frac{2mE}{\hbar^2}$

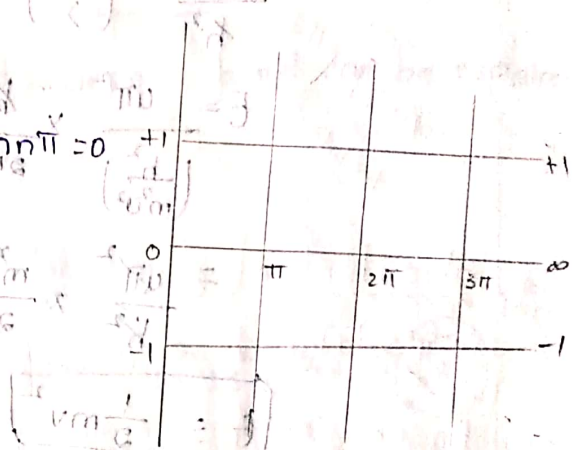
$$\therefore \frac{2mE}{\hbar^2} = \frac{n^2 \pi^2}{a^2}$$

$$E = \frac{n^2 \pi^2}{a^2} \times \frac{\hbar^2}{2m}$$

$$= \frac{n^2 \pi^2}{2ma^2} \times \frac{\hbar^2}{4\pi^2}$$

$$\therefore E = \frac{n^2 \hbar^2}{8ma^2}$$

$n = 1, 2, 3, \dots$



5. If the scattering power of the barrier  $p \rightarrow 0$  then the forbidden band will disappear and the particle will be free in b/w  $\pm 1$  as shown in the figure.



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from eqn (1)  $P=0$

then,  $\cos \alpha a = \cos ka$   
 $\alpha a = ka$

$\alpha = \frac{ka}{a}$

$\alpha = k$

$\frac{2mE}{\hbar^2} = \left(\frac{2\pi}{\lambda}\right)^2$

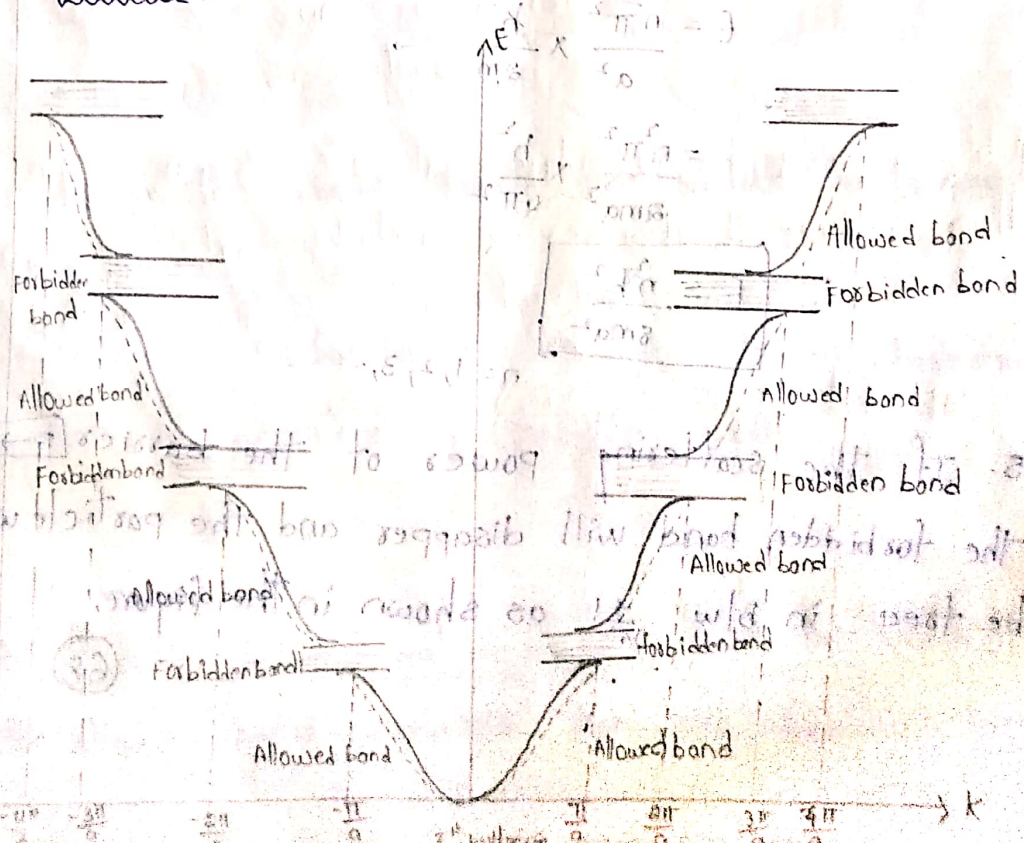
$E = \frac{4\pi^2}{\left(\frac{h^2}{m^2 v^2}\right)} \times \frac{\hbar^2}{2m}$

$= \frac{4\pi^2}{\hbar^2} \times \frac{m^2 v^2}{2m} \times \frac{\hbar^2}{4\pi^2}$

$E = \frac{1}{2} m v^2$

The above equation represents K.E of free particle.

E-k diagram (or) Brillouin zones:



→ Brillouin showed that the allowed regions and forbidden regions is in the form of zones which are called Brillouin zone

→ These are shown on the graph between  $E$  and  $k$

→ According to him the first Brillouin zone is  $-\frac{\pi}{a} \rightarrow \frac{\pi}{a}$

The 2<sup>nd</sup> Brillouin zone is  $\left\{ -\frac{2\pi}{a} \rightarrow -\frac{\pi}{a} \right\}$  and  $\left\{ \frac{\pi}{a} \rightarrow \frac{2\pi}{a} \right\}$

The 3<sup>rd</sup> Brillouin zone is  $\left\{ -\frac{3\pi}{a} \rightarrow -\frac{2\pi}{a} \right\}$  and  $\left\{ \frac{2\pi}{a} \rightarrow \frac{3\pi}{a} \right\}$

origin of formation of energy bands:-

The origin of formation of energy band can be explained by using of example.

ex:- carbon  
( $Z=6$ )

Electronic configuration is  $1s^2 2s^2 2p^2$

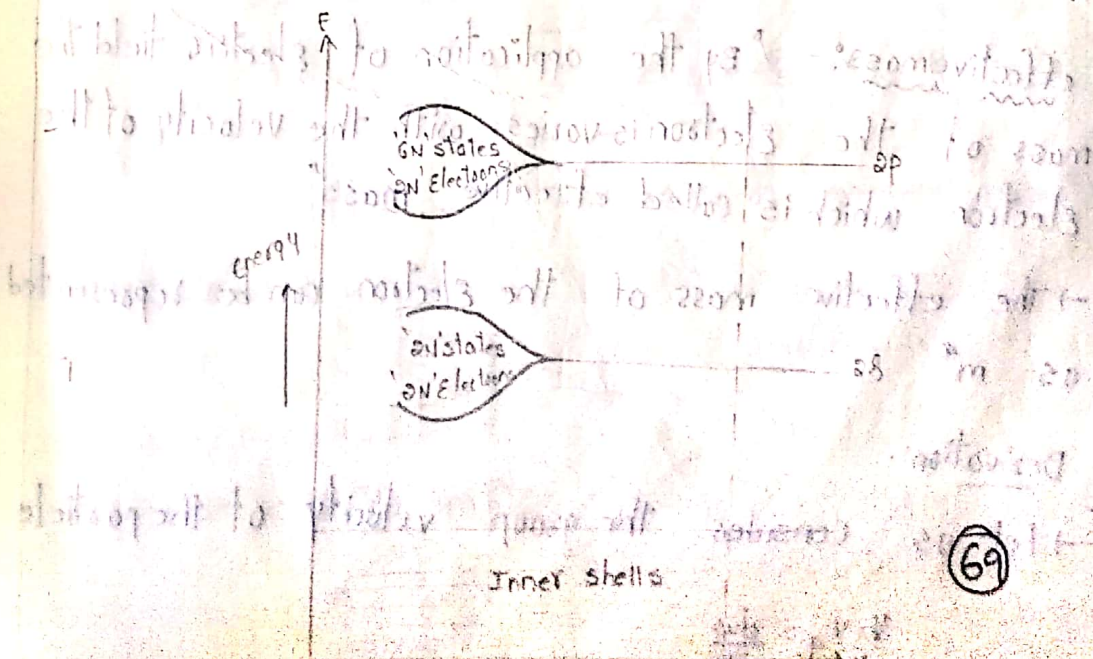


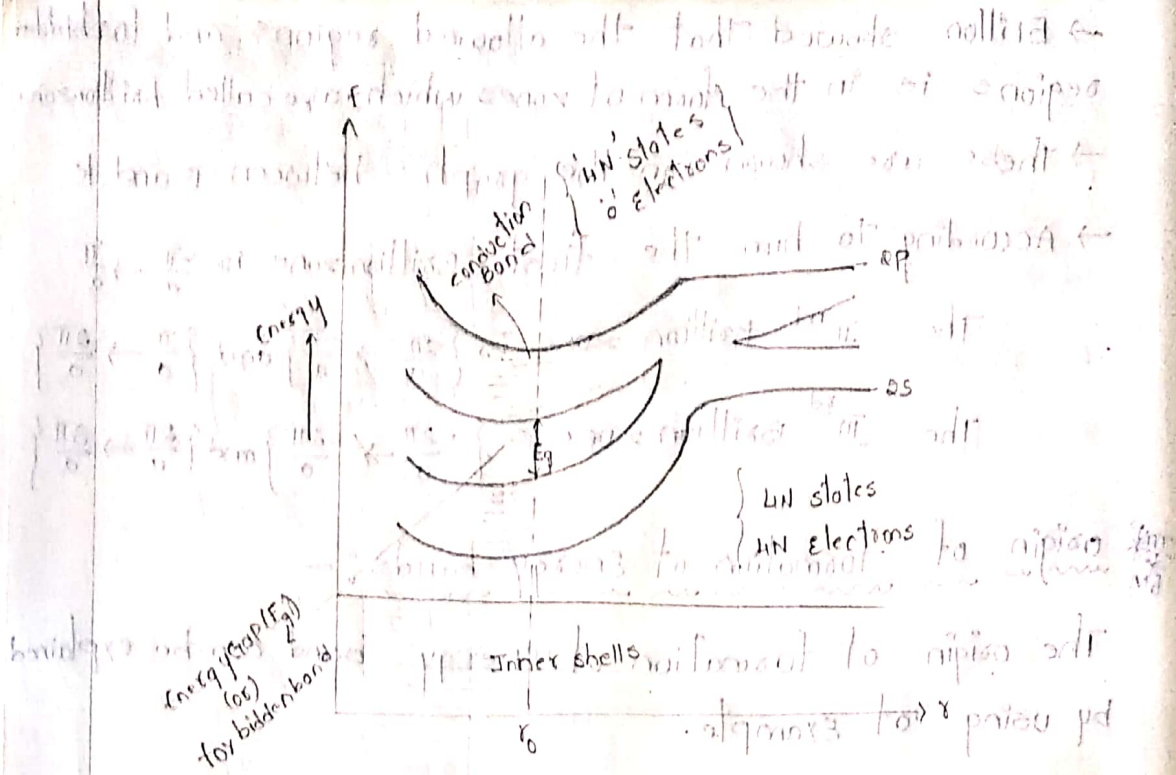
i.e., the outermost orbit electrons in carbon is 4.

The valency of carbon is 4.

→ The orbital representation diagram of carbon is as shown in the figure.

If suppose 'N' be the no. of atoms in the carbon element then the orbital representation is as shown in the figure.





\* Imp classification of materials into conductors, semi-conductors & insulators:-

Based on the energy gap between the conduction band and valency band the solids are classified into three types.

They are:

1. conductors.
2. semi-conductors.
3. Insulators.

effective mass:- "By the application of electric field the mass of the electron is varies with the velocity of the electron which is called effective mass".

→ The effective mass of the electron can be represented as  $m^*$ .

Derivation:-

→ let us consider the group velocity of the particle

$$v = v_g = \frac{d\omega}{dk}$$

$$\left[ v = \frac{d\omega}{dk} \right] = 0$$

where,

$\omega \rightarrow$  angular frequency

$$\boxed{\omega = 2\pi\nu}$$

from (1)  $\Rightarrow v = \frac{d(2\pi\nu)}{dk}$  — (2)

$$E = h\nu \Rightarrow \boxed{\nu = \frac{E}{h}}$$

from (2)

$$v = \frac{d}{dk} \left( \frac{2\pi}{h} \right) E$$

$$\boxed{v = \frac{1}{h} \frac{dE}{dk}}$$
 — (3)

Now differentiate above equation w.r.t 't'

$$\frac{dv}{dt} = \frac{d}{dt} \left[ \frac{1}{h} \frac{dE}{dk} \right]$$

$$a = \frac{1}{h} \left[ \frac{d^2 E}{dk dt} \right]$$

$$a = \frac{1}{h} \frac{d^2 E}{dk dt} \left[ \frac{dk}{dk} \right]$$

$$a = \frac{1}{h} \frac{d^2 E}{dk^2} \left[ \frac{dk}{dt} \right]$$
 — (4)

We know that  $\boxed{p = \hbar k}$  — (5)

Now differentiating above equation w.r.t to 't'

$$\boxed{\frac{dp}{dt} = \hbar \frac{dk}{dt}}$$
 — (6)

We know that, the rate of change of momentum is called force.

$$\boxed{\frac{dp}{dt} = F}$$
 — (7)

sub (7) in (6)

$$F = \hbar \frac{dk}{dt}$$

$$\boxed{\frac{dk}{dt} = \frac{F}{\hbar}}$$
 — (8)

(70)



sub (8) in (4)

$$a = \frac{1}{\hbar} \left[ \frac{d^2 E}{dk^2} \right] \frac{F}{\hbar}$$

$$a = \frac{F}{\hbar^2} \left[ \frac{d^2 E}{dk^2} \right] \quad \text{--- (9)}$$

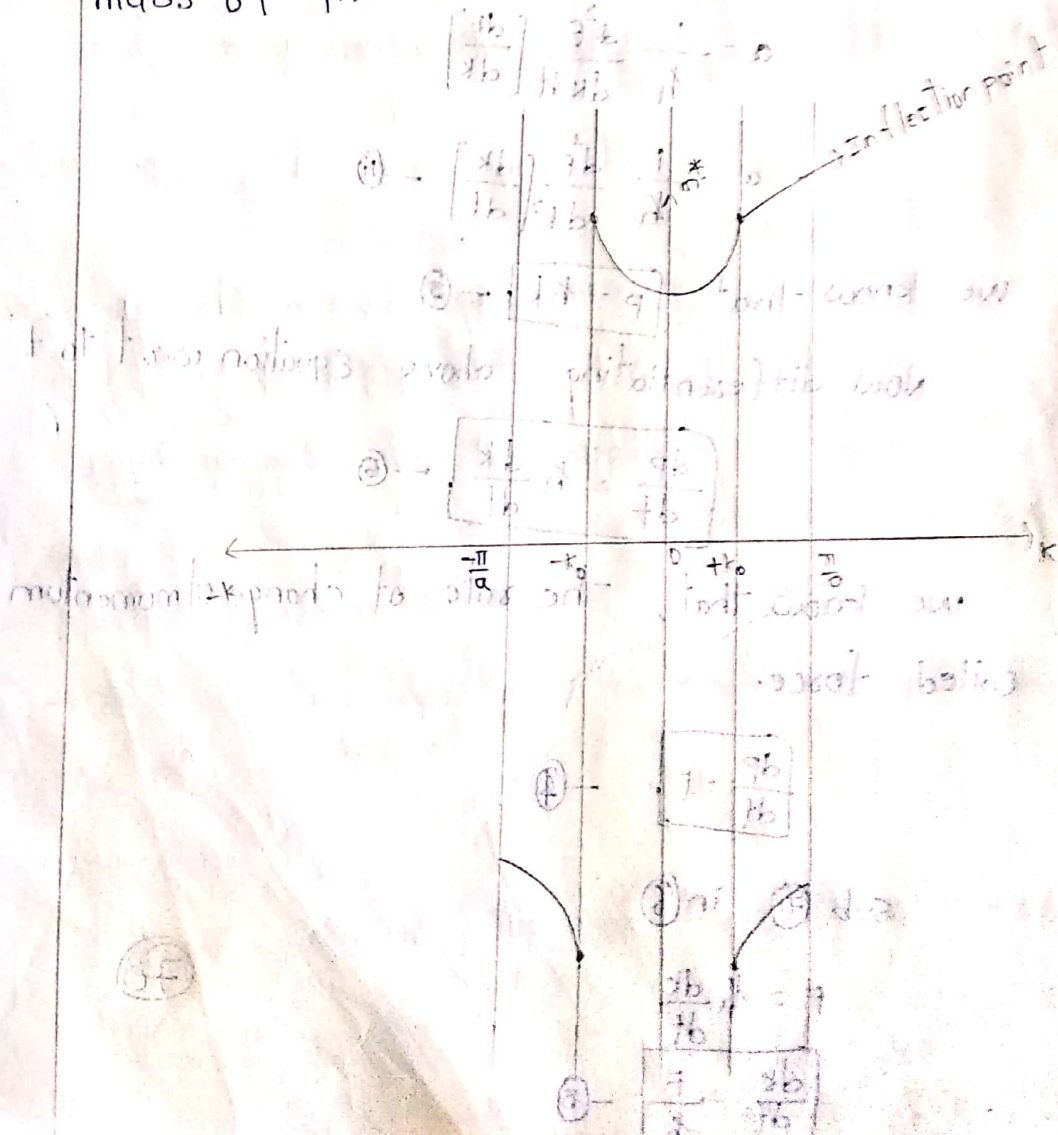
$$F = m^* a \quad \text{--- (10)}$$

sub (10) in equ (9)

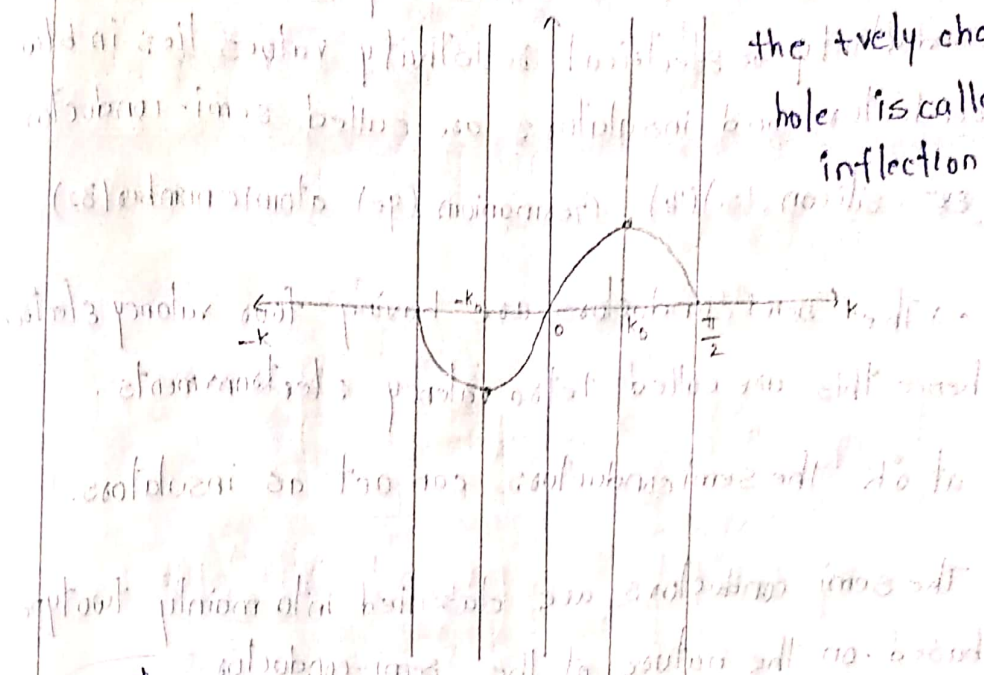
$$a = \frac{m^* a}{\hbar^2} \left[ \frac{d^2 E}{dk^2} \right]$$

$$m^* = \frac{\hbar^2}{\left[ \frac{d^2 E}{dk^2} \right]}$$

∴ The above equation represents the effective mass of the electrons



→ The point at which the -vely charged electron can behave as the +vely charged hole is called inflection point.



Questions:-

1. State  $\epsilon_{\text{effective}}$  effective mass of electron and explain the concept of hole?
2. Explain the origin of formation of energy bands in solids?
3. Differentiate (or) classify the materials into conductors, semi-conductors and insulators?
4. Explain the Kronig-penney model (or) Explain the periodic motion of electron in atomic lattice?
5. State & Explain the Bloch theorem?

→ The mass of the electron is a function of velocity. By the application of electric field the mass of the electron is also increasing with velocity 'v' upto a point 'k<sub>0</sub>'. (71)  
 → Beyond the point k<sub>0</sub> the velocity of the particle reduces and it will get retardation. (-ve acceleration) i.e; the negatively charged electron can behave as the positively charged 'hole', which is the concept of hole.